

# Application of Classical and Output-Only Modal Analysis to a Laser Cutting Machine

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## Abstract

This paper presents the application of classical modal analysis and output-only modal analysis techniques (Stochastic Subspace Identification and Frequency Domain Decomposition) to a laser cutting machine. It will be shown that the classical modal analysis gave fundamental insight into the structural dynamics of the laser cutting machine, while the output-only modal analysis from operational vibration data provided the modes of vibration directly influencing the cutting results. From the operational data an additional mode of vibration was identified that could not be extracted by the utilized classical modal analysis techniques. It was revealed that this mode of vibration was nonstructural and that it was caused by the drives and the control system of the laser cutting machine. Furthermore, this nonstructural mode could directly be linked to imperfections of the cutting results for specific machine movements. With this knowledge a cost effective and easy to implement solution was developed that significantly enhanced the quality of the cutting results.

## 1 Introduction

For laser cutting medium to large sized steel sheets (e.g. 3 by 1.5 m) are usually processed. The steel sheets are placed on a horizontal grate and the laser cutting head is moving in-plane along the cutting edges (principle of 'flying optics').

Today, high dynamic performance is desired for laser cutting machines leading to lightweight machine constructions. Accelerations of more than 1 g are typically realized in order to keep auxiliary process times as short as possible. On the other hand, steel sheets of up to 25 mm thickness are processed. This requires a very slow and precise movement of the laser cutting head in order to achieve maximum fidelity with respect to form and tolerance (typically  $\pm 0.05$  mm) of the workpiece.

The two highlighted extremes of high dynamic performance on one side and very accurate slow movements on the other, demand for supreme harmonization of drives, control system, and, of course, of the mechanical machine construction.

One specific goal for laser cutting machines is that the influence of the structural modes of vibration will not degrade the high quality of the cutting results. If this goal cannot be met under certain operating conditions, the critical modes of vibration must be identified and remedial measures are to be developed.

This paper presents an example of a laser cutting machine where a specific dynamic movement caused slight imperfections of the cutting results. In order to improve this situation classical modal analysis and output-only modal analysis techniques (Frequency Domain Decomposition and Stochastic Subspace Identification) were applied to assess the dynamic behavior of the laser cutting machine. It will be shown that the classical modal analysis gave fundamental insight into the structural dynamics of the laser cutting machine, while the output-only modal analysis from operational vibration data provided the modes of vibration directly influencing the cutting results.

From the operational data an additional mode of vibration was identified that could not be extracted by the classical modal analysis techniques. A subsequent investigation revealed that this mode of vibration was nonstructural and was caused by the drives and the control system of the laser cutting machine.

Finally, the nonstructural mode of vibration could directly be linked to the observed imperfections of the cutting results. With this knowledge a cost effective and easy to implement solution was developed that significantly enhanced the quality of the cutting results.

## 2 Theory Overview

In the following the theory of the applied output-only modal analysis techniques will be summarized. The classical experimental modal analysis techniques are well known and will not be presented. An introduction to these techniques can for instance be found in references [1-3].

### 2.1 Frequency Domain Decomposition

The Frequency Domain Decomposition technique described in reference [4] is an extension of the classical frequency domain approach referred to as the Basic Frequency Domain or Peak Picking technique. For the classical technique modes of vibration are directly extracted from the power spectral density matrix of the measured responses at the resonance peaks [5], while it is required that the modes of vibration are well separated. Of course, this is not always the case.

The power spectral density matrix, that can be estimated easily for instance via Fast Fourier Transformation, is also utilized by the Frequency Domain Decomposition technique. However, in contrast to the classical technique, it is not directly processed, but decomposed by applying the Singular Value Decomposition at each spectral line. Proceeding this way decomposes the power spectral density matrix into auto spectral density functions of single degree of freedom systems.

Frequencies and damping values can subsequently be extracted from the auto spectral density functions of the single degree of freedom systems at the resonance peaks, while the corresponding modes of vibration are estimated directly from the singular value vectors.

For the Frequency Domain Decomposition technique the following three assumptions must be met:

- 1) the loading must be white noise
- 2) the structure must be lightly damped
- 3) closely spaced modes of vibration must be geometrically orthogonal

If the above assumptions cannot be satisfied the decomposition of the power spectral density matrix into auto spectral density functions of single degree of freedom systems is not exact. Thus the results of the Frequency Domain Decomposition technique are merely approximate in this case, but can be expected to be more accurate than the results from the classical technique.

### 2.2 Stochastic Subspace Identification

Stochastic Subspace Identification techniques are focusing on the discrete time state space model of a linear, time invariant system without external inputs according to equation (1), see also reference [6].

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{v}_k \end{aligned} \quad (1)$$

with:

- k** index for time instant  $k$
- x** state vector (not measurable)
- y** output vector (measurable)
- w, v** zero mean, stationary white noise sequences representing process and measurement noise respectively (not measurable)
- A** (dynamical) system matrix completely characterizing the dynamics of the system
- C** output matrix transforming the state vector onto the output vector

In order to identify modal data Stochastic Subspace Identification techniques are in a first step estimating the matrices **A** and **C** solely from the measured output in **y**. Basically this can be achieved by building up an output block Hankel matrix, which holds all information of the correlation functions of the measured responses. The block Hankel matrix is factorized utilizing a Singular Value Decomposition that is truncated to the selected model order, and from the factorization results the desired matrices **A** and **C** are obtained. In a second step, the modal frequencies, damping values, and modes of vibration are directly extracted from the estimated system matrix **A**.

One important issue for Stochastic Subspace Identification techniques is the choice of the model order, i.e. the size of the system matrix **A** or the state vector **x** respectively. To assist in finding the appropriate model order, stabilization diagrams - as known from classical experimental modal analysis techniques - may be utilized.

## 3 Application

### 3.1 Objective

For a specific type of laser cutting machine it was observed that, when cutting circular workpieces with diameters ranging from 200 to 400 millimeters and utilizing a particular set of process parameters, slight waveform like imperfections occur along the cutting edge (figure 1).

From the geometry of the imperfection (wavelength) and the process parameters (feeding speed) it was concluded, that a mode of vibration at about 18 Hz is relevant.



Figure 1: Example of observed imperfections

In order to locate the source of the unwanted machine behavior multiple tests of structural components, control system modifications, software changes, and FEM calculations were conducted. All efforts indicated a complex interaction of multiple parameters, but no weak points in the machine design could be revealed.

Thus, to improve the described situation two additional test campaigns were defined:

- 1) a classical modal test and modal analysis in order to obtain fundamental insight into the structural dynamics of the laser cutting machine
- 2) an in-operation test with subsequent output-only modal analysis in order to assess the dynamic behavior of the laser cutting machine under the critical process conditions

### 3.2 Classical Modal Analysis

#### 3.2.1 Modal Test

To determine the structural modes of vibration of the laser cutting machine, a modal test was performed. The machine axes (cutting bridge and cutting head) were set to mid positions, and the drives plus the control system were activated in order to

maintain the selected position of the machine axes during the test.

The excitation of the laser cutting machine was provided by a mid sized impact hammer in two different directions (two references, one horizontal and one vertical, see figures 2 and 3).



Figure 2: Impulse excitation of machine

Roving accelerometer testing was selected. It was favored over roving hammer testing, because the excitation level can be kept more or less the same for all measurement sets to be processed. This bears the advantage that possible non-linearities will not be excited in a significantly changing manner from measurement set to measurement set.

Accelerations were measured with 15 uni-axial accelerometers simultaneously utilizing a DIFA SCADAS 2 front-end and IDEAS data acquisition software. Because a total of 77 measurement degrees of freedom at 50 individual measurement points had to be processed (figure 3), six different measurement sets were taken.

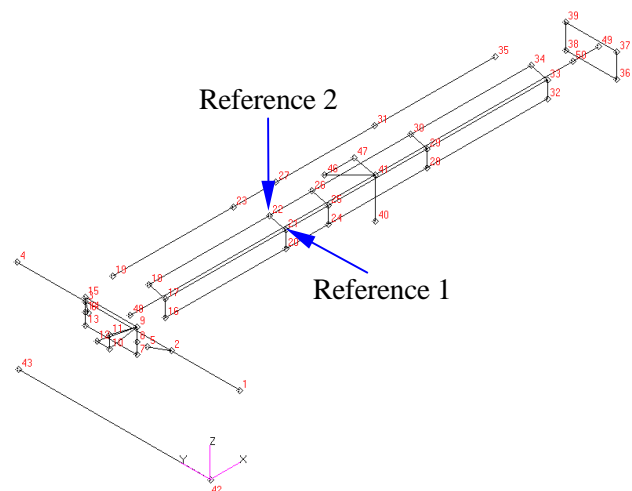


Figure 3: Measurement points and references

### 3.2.2 Modal Identification

The modal identification was performed utilizing well established curve fitting and direct parameter estimation techniques and yielded 66 modes of vibration in the frequency range from zero to 100 Hz.

In order to obtain a qualitative impression, the modes of vibration in the vicinity of the major resonance peaks were investigated. Figure 4 and table 1 give an overview of the corresponding mode of vibration classes. An example for the most significant mode of vibration class in the frequency range around 25.0 Hz (range 2) can be found in figure 5.

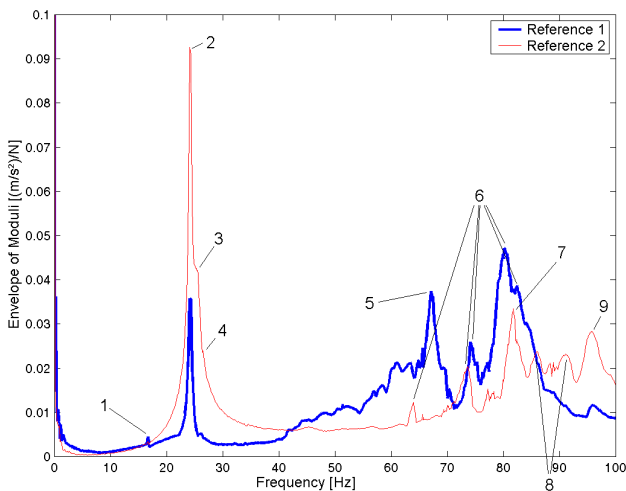


Figure 4: Envelope of frequency response moduli

Table 1: Classification of modes of vibration

Range	Description
1	bending of driving shaft of cutting bridge
2	coupled bending in Y/Z of cutting bridge
3	coupled torsion about X/bending about Z of cutting bridge
4	translation in X of cutting bridge
5	mirrors, local
6	ditto
7	coupled bending in Y/Z of cutting bridge
8	ditto
9	coupled bending in Y/Z of cutting bridge + support in X

With respect to the critical frequency causing the imperfections (about 18 Hz) it can be stated, that neither the bending frequencies of the driving shaft of the cutting bridge (range 1 in figure 4, about 16.5 Hz) nor the coupled bending frequencies in Y/Z of the cutting bridge (range 2 in figure 4, about 25.0 Hz) coincide. I.e. no structural mode of vibration could be identified at the critical frequency.

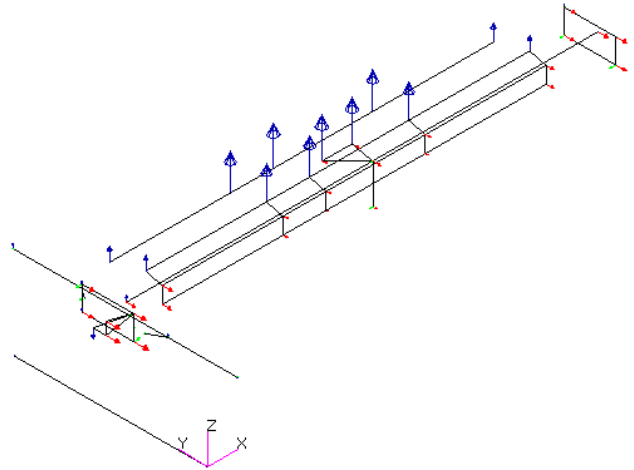


Figure 5: Mode of vibration at 25.0 Hz

### 3.3 Output-Only Modal Analysis

#### 3.3.1 In-Operation Test

In order to assess the relevant vibrations of the laser cutting machine in operation the critical cutting movement was simulated (the laser was not active during the measurements in order not to damage the measurement equipment).

Accelerations were measured again with 15 uni-axial accelerometers simultaneously utilizing a DIFA SCADAS 2 front-end and IDEAS data acquisition software (figure 6). Because a total of 20 measurement degrees of freedom at 17 individual measurement points had to be processed, two different measurement sets were taken. For the two different measurement sets 10 measurement degrees of freedom were measured redundantly. This was required in order to assemble the overall modes of vibration subsequently.



Figure 6: In-operation test setup

In figure 7 a typical time record of the cutting process is given. Taking a look at a zoom into this time record (figure 8) exhibits that a dominant frequency at about 18 Hz is present, i.e. at the critical frequency.

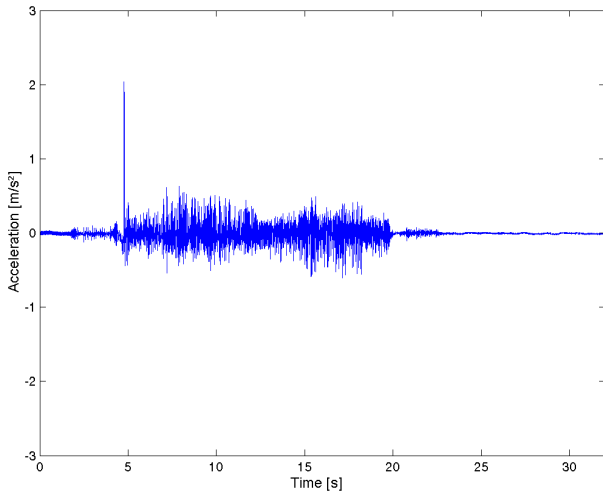


Figure 7: Typical time record

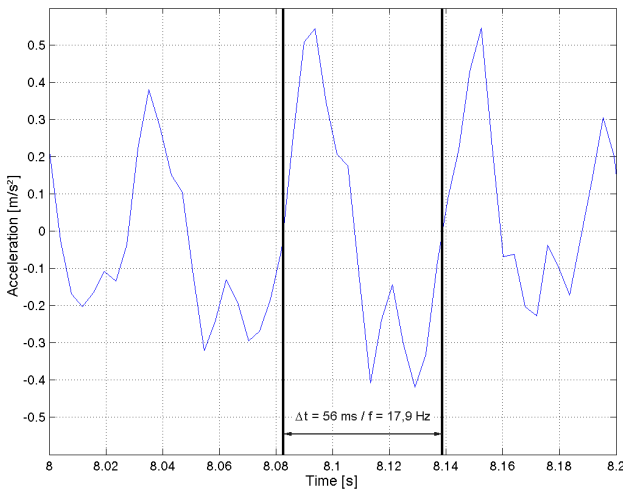


Figure 8: Typical time record (zoomed)

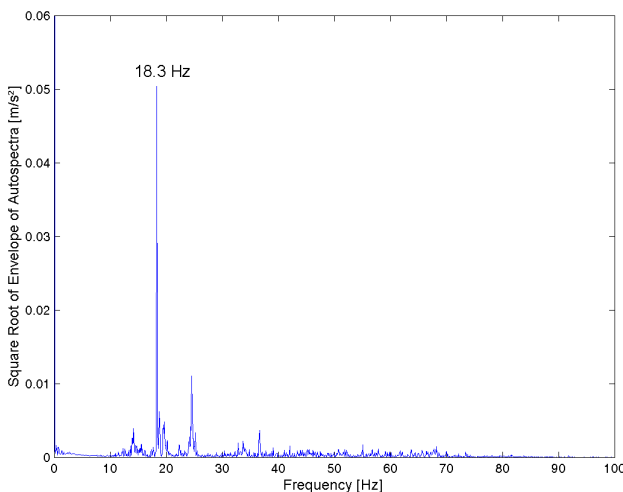


Figure 9: Frequency content of cutting process

Figure 9 shows the square root of the envelope of estimated autospectra, revealing the frequency content of the cutting process. Again, a dominant response at 18.3 Hz can be observed (at the critical frequency).

### 3.3.2 Output-Only Modal Identification

To identify the operational modes of vibration governing the response of the laser cutting machine at about 18 Hz, output-only modal analysis techniques were applied utilizing special ICS in-house software. Two different techniques were applied:

- 1) Frequency Domain Decomposition
- 2) Stochastic Subspace Identification

For both techniques time data were measured. The data were subsequently digitally band-pass filtered from 10 to 30 Hz in order to focus exclusively on the critical frequency range.

For the Frequency Domain Decomposition the spectral density matrix was calculated utilizing a 2048 point Fast Fourier Transformation. In figure 10 the singular value decomposition of the spectral density matrix is shown for the first measurement set, and modes of vibration can be recognized at about 18 and 25 Hz.

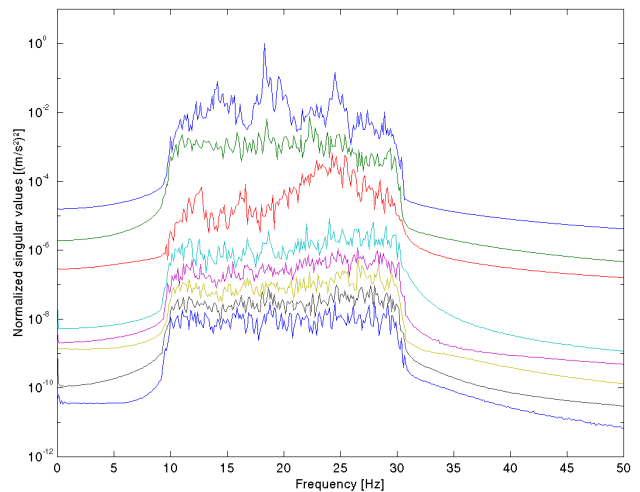


Figure 10: Singular value decomposition of spectral density matrix

For the Stochastic Subspace Identification 3800 data values were used to calculate the output block Hankel matrix. Figure 11 shows the stabilization diagram for the first measurement set. It can be noticed that no fully satisfactory stabilization occurs. However, at about 14, 18, and 25 Hz relatively stable modes of vibration are present. The final model order selected for modal data extraction was 10. Only the

modes of vibration at about 18 and 25 Hz were retained since the other ones exhibited unreasonable damping values.

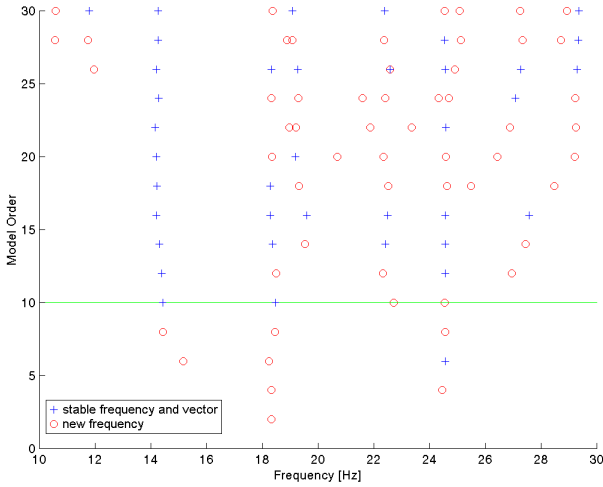


Figure 11: Stabilization diagram

All in one both techniques yielded the same two modes of vibration (see table 2 and figures 12 and 13), while the agreement of the individual results was very good.

Table 2: Identified operational modes of vibration

No.	Freq. [Hz]	Description
1	18.3	translation in Y of cutting bridge
2	24.5	coupled bending in Y/Z of cutting bridge

The overall operational modes of vibration were assembled from the two different measurement sets utilizing the 10 redundantly measured degrees of freedom. A modal scale factor was determined according to equation (2) for each pair of modes of vibration in order to achieve the same scaling (within least squares limits).

$$MSF = \frac{\mathbf{x}_{Test2}^T \mathbf{x}_{Test1}}{\mathbf{x}_{Test2}^T \mathbf{x}_{Test2}} \quad (2)$$

with:

$\mathbf{x}_{Test1}$  mode of vibration from measurement set 1 at the 10 redundantly measured degrees of freedom

$\mathbf{x}_{Test2}$  mode of vibration from measurement set 2 at the 10 redundantly measured degrees of freedom

The additional five measurement degrees of freedom from the second measurement set were scaled with the corresponding modal scale factors

and then simply added to the 15 measurement degrees of freedom obtained from the first measurement set.

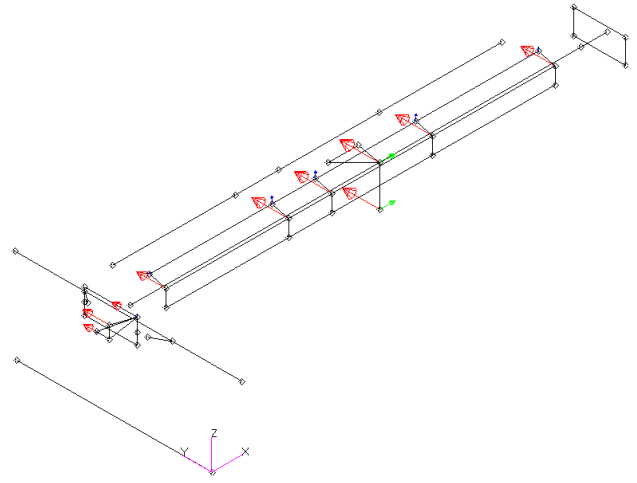


Figure 12: Operational mode of vibration at 18.3 Hz

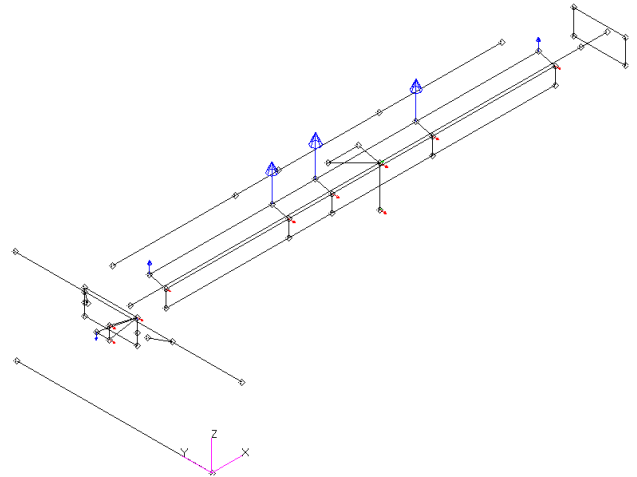


Figure 13: Operational mode of vibration at 24.5 Hz

### 3.4 Comparison of Results

A comparison of the two operational modes of vibration and the 66 modes of vibration identified by classical experimental modal analysis techniques reveals the following:

- 1) the operational mode of vibration at 24.5 Hz correlates very well with an experimental modal analysis mode of vibration at 25.0 Hz (MAC value of 93.0 %)
- 2) the operational mode of vibration at 18.3 Hz cannot be paired to any experimental modal analysis mode of vibration; it is obviously non-structural and is most probably caused by the drives and the control system

### 3.5 Machine Improvement

Based on the assembled knowledge, especially from the output-only modal analysis, it was possible to exactly locate the critical parts of the laser cutting machine: Here, for a special combination of process parameters, the drives and the control system introduced an additional nonstructural mode of vibration that caused the observed imperfections.

Considering this next to the structural modes of vibration identified from the classical modal analysis an appropriate software update for the control system was developed, that entirely eliminated all critical frequencies from the cutting process.

The described action lead to a significant performance improvement of the laser cutting machine. In particular the required fidelity with respect to form and tolerance ( $\pm 0.05$  mm) of the workpiece can now be realized.

## 4 Summary

This paper presented an example of a laser cutting machine where a specific dynamic movement caused slight imperfections of the cutting results. In order to improve this situation classical modal analysis and output-only modal analysis techniques were applied to assess the dynamic behavior of the laser cutting machine.

The classical modal analysis yielded 66 modes of vibration and gave fundamental insight into the structural dynamics of the laser cutting machine. The output-only modal analysis from operational vibration data provided two modes of vibration that directly influenced the cutting results.

From the operational data an additional mode of vibration was identified that could not be extracted by classical modal analysis techniques. It showed that this mode of vibration was nonstructural and was caused by the drives and the control system of the laser cutting machine.

Finally, the nonstructural mode of vibration could directly be linked to the observed imperfections of the cutting results. With this knowledge a software update for the control system was developed that significantly enhanced the quality of the cutting results.

The presented example demonstrates that the classical techniques utilized for the development of machine tools have their limitations. Particularly if high dynamic performance is demanded, an integrated application of classical and output-only modal analysis can help to pinpoint critical machine

parts and to develop remedial measures in an efficient, cost effective way.

## Acknowledgements

The authors like to thank Mr. Tim Dannat of MTS Germany for the good cooperation with respect to all IDEAS related data acquisition and post test analyses issues.

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