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PARAMETER IDENTIFICATION AND VALIDATION OF LARGE ORDER FINITE ELEMENT MODELS FOR INDUSTRIAL TYPE STRUCTURES

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This lecture gives a brief overview of procedures for validating analytical models using experimental data including some exemplary results. The presentation is restricted to current procedures established by experience with large order industrial applications. The basic numerical techniques for updating the parameters of Finite Element (FE) models are described. Sources of modeling and test data uncertainties are addressed together with related requirements concerning the quality of the initial analytical model and the test data. Obstacles to ensure the prediction capability of the updated models to untested situations are discussed. The described applications include an automotive car body and a civil aircraft structure.

Keywords: *Model updating, model validation, experimental modal analysis, parameter identification*

1 Introduction

Considerable discrepancies between Finite Element (FE) structural analysis results and experimental data are often observed in complex practical applications. The classical way to reduce these discrepancies is to modify by trial and error the assumptions made for the mechanical idealizations and the parameters of the analytical model until the correlation of analytical predictions and experimental results satisfies practical requirements. For complex FE models this trial and error approach turns out to be very time consuming and sometimes is not feasible at all. Some effort has therefore been spent in the past in the development of computational procedures for updating the parameters of analytical models using dynamic test data (computational model updating, CMU).

The expected non-uniqueness of the updated models due to different CMU methods, different structural idealizations, different parameter sets and, of course, different test data sets being polluted by unavoidable noise and not being representative for the whole sample space can be tolerated if the updated models retain their prediction capability with regard to the intended purpose. In this case, and only in this case, an updated model may be considered as an equivalent model validated with respect to a special purpose. The problem remains how to check the prediction capability.

The following requirements for a validated model were proposed within the European research co-operation COST, ref. [1], some of which were also applied for the application examples described below:

1) The equivalent model must be capable of predicting the experimental modal data and/or the frequency response functions (FRFs) within the active frequency range and within certain accuracy limits. The term active frequency range is related to the frequency range used for computational model updating. This criterion represents a minimum requirement which does not yet say much about the prediction quality of the model. The prediction quality should therefore be checked using the following additional criteria:

2) Prediction of the eigenfrequencies and modes beyond the active frequency range (passive range).

3) Prediction of the frequency response functions (FRFs) obtained from loading conditions other than those used for CMU.

4) Prediction of the modal data and/or FRFs of a modified structure. The structural modification might consist of one or more added masses or by changed boundary conditions.

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In this lecture a brief overview of methods and results of using computational optimization techniques to update the parameters of large order FE models is presented together with assessments of the model validity depending on the final utilization purpose. The described applications include the results of an automotive car body and a civil aircraft structure.

2 Theoretical Background

Computational model updating (CMU) procedures are aimed at fitting selected model parameters such that the test/analysis deviations (residuals) are minimised. The residuals $\Delta\mathbf{z} = \mathbf{z}_T - \mathbf{z}(\mathbf{p})$ ($\mathbf{z}_T$:= test data vector, $\mathbf{z}(\mathbf{p})$ =: corresponding analytical data vector) usually depend in a non-linear way on the parameters. Thus the minimization problem is also non-linear and must be solved iteratively. One way is to use the classical sensitivity approach (e.g. references [2-4]) where the analytical data vector is linearised by a Taylor series expansion truncated after the first term which leads to:

$$\Delta\mathbf{z} = \Delta\mathbf{z}_0 - \mathbf{G}_0\Delta\mathbf{p} \qquad (1)$$

with

$\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$ design parameter changes,

$\mathbf{p}_0$ design parameter vector at linearization point (index “0”),

$\Delta\mathbf{z}_0 = \mathbf{z}_T - \mathbf{z}(\mathbf{p}_0)$ test/analysis differences at linearization point, for example, the differences between eigenfrequencies, mode shapes or frequency responses,

$G_0 = \partial\mathbf{z}/\partial\mathbf{p}\big|_{\mathbf{p}=\mathbf{p}_0}$ sensitivity matrix at linearization point.

The desired model parameter changes $\Delta\mathbf{p}$ are obtained by minimizing the following objective function J with respect to the parameter changes $\Delta\mathbf{p}$:

$$J(\Delta\mathbf{p}) = \Delta\mathbf{z}^T\mathbf{W}\Delta\mathbf{z} + \Delta\mathbf{p}^T\mathbf{W}_p\,\Delta\mathbf{p} \rightarrow \min \qquad (2)$$

where $\mathbf{W}$ and $\mathbf{W}_p$ are weighting matrices. The second term in equation (2) is used to constrain the parameter variation. The minimization condition $\partial J/\partial\Delta\mathbf{p} = 0$ yields the linear equation system (3) which has to be solved with respect to the vector of parameter changes $\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$ at the linearization point.

$$\left(\mathbf{G}_0^T\mathbf{W}\mathbf{G}_0 + \mathbf{W}_p\right)\Delta\mathbf{p} = \mathbf{G}_0^T\mathbf{W}\Delta\mathbf{z}_0 \ . \qquad (3)$$

The iterative numerical procedure starts from nominal parameter values which are updated after each iteration step: $\mathbf{p}_0 \Rightarrow \mathbf{p}_0 + \Delta\mathbf{p}$. The parameter vector $\mathbf{p}_0$ from the previous iteration step represents the linearization point. Depending on the choice of the residuals used in the objective function the sensitivity matrix may include the sensitivities $\partial\boldsymbol{\lambda}/\partial p_i$ ($\boldsymbol{\lambda}$ = vector of eigenvalues), $\partial\boldsymbol{\varphi}/\partial p_i$ ($\boldsymbol{\varphi}$ = mode shapes), $\partial\mathbf{u}/\partial p_i$ ($\mathbf{u}$ = vector of frequency responses) or other residuals with respect to the i-th parameter p_i.

For $\mathbf{W}_p$= $\mathbf{0}$, equation (3) represents a standard weighted least squares problem.

The above minimization procedure requires a parameterization of the model with respect to the stiffness, mass and damping parameters, $\mathbf{p}$, in the equation of motion,

$$(-\omega^2\mathbf{M}(\mathbf{p}) + j\omega\mathbf{D}(\mathbf{p}) + \mathbf{K}(\mathbf{p})\,)\mathbf{u} = \mathbf{F} \qquad (4)$$

($j = \sqrt{-1}$, ω = excitation frequency, $\mathbf{u}$ = complex frequency response vector and $\mathbf{F}$ = excitation force vector).

The vector $\mathbf{p}$ contains the unknown correction parameters addressing the uncertain parameters of the stiffness matrix **K**, the mass matrix **M**, and the damping matrix **D**. The type and the location of these parameters must be specified by the user which, for complex practical applications, often turns out to be a bigger problem than calculating their values by numerical procedures. The sensitivities can directly be calculated by finite differences in each iteration step, for example by:

$$\partial\lambda/\partial p_i = \left(\ \lambda\left(p_i + \varepsilon p_i\right) - \lambda(p_i)\ \right)/\varepsilon p_i$$
$$\textit{...others...} \qquad (5)$$

(ε = small perturbation number, i =1,..., np = no. of parameters to be updated).

This formulation allows for updating geometrical shape parameters like grid point locations and even the FE mesh density, ref. [5]. Non- linear model parameters can also be updated as was shown in refs. [6]. In this case the non- linear time domain equation of motion was transformed to the frequency domain by using the harmonic balance theory which leads to the form

$$[-\omega^2\mathbf{M} + j(\omega\mathbf{D} + \mathbf{D}^{NL}(u)\,) + \mathbf{K} + \mathbf{K}^{NL}(u)]\,\mathbf{u} = \mathbf{F} \, . \qquad (6)$$

where $\mathbf{D}^{NL}(u)$ and $\mathbf{K}^{NL}(u)$ represent non- linear terms depending on the displacement amplitudes.

The variance of the test data can be taken into account by proper assumptions for the weighting matrix **W** in eq.(2). It can be shown by linear statistical analysis that the covariance matrix of the parameter estimate is related to the covariance matrix of the test data vector by

$$\mathrm{cov}(\Delta\mathbf{p}) = \mathbf{G}^+\,\mathrm{cov}(\mathbf{z}_T)\,\mathbf{G}^{+T} \qquad (7)$$

where $\mathbf{G}^+$ denotes the pseudo inverse of the sensitivity matrix. This equation illustrates the importance of the condition of the sensitivity matrix which governs the covariance of the parameter estimate. The condition of the sensitivity matrix is governed by the type and the number of parameters selected for updating. An unfavorable parameter selection may thus lead to meaningless parameter estimates.

Eq.(7) assumes that the sensitivity matrix represents a deterministic quantity. This assumption is in contrast to reality since the sensitivity matrix is also a function of the model parameters.

Procedures to take into account both the scatter of the test data as well as the scatter of the parameters are being developed at present and form a future direction in model updating and validation. Monte Carlo methods and the stochastic finite Element method introduced in ref. [7] may be used to simulate the response of the model due to stochastic modeling parameters.

Other techniques being under development are based on the assumption that test data and model parameters may be considered as fuzzy numbers which do not necessarily have to involve assumptions on statistical properties, ref. [8]. Instead membership functions are defined which denote the grade of membership of a parameter in a fuzzy subset.

The problem to solve the inverse fuzzy arithmetic problem, given the test data as fuzzy numbers and getting back the parameter values as fuzzy numbers, is at the beginning and represents a promising research direction. An example can be found in ref. [9].

3 Applications

3.1 Automotive Car Body

A study supported by the German Automotive Industry, Working Group Structural Optimization and Acoustics, ref. [10], was aimed at validating the Finite Element model of a car body in white as shown in fig.1.

For keeping the inevitable uncertainties from the test side as small as possible a thorough test planning is essential. Utilizing the initial Finite Element model both measurement degrees of freedom and exciter positions are first determined based on the results of a numerical algorithm established on a linear independence criterion, ref. [11].

For complex industrial applications it may happen that computational model updating will not yield physically meaningful results for all chosen parameters in the first place. In such cases areas of the investigated system where severe modeling deficiencies exist can often be identified by sensitivity analysis, which forms an integral part of the updating algorithm. Dedicated remodeling of such areas can usually improve the situation, leading to a better basis for computational model updating.

Subsequent computational model updating may then lead to an improved correlation and a noticeable reduction of the test/analysis deviations over a broad frequency range. Fig. 2 for example shows the eigenfrequency deviations and MAC values describing the correlation of the mode shapes before and after computational model updating (MAC = 100% would mean perfect correlation).

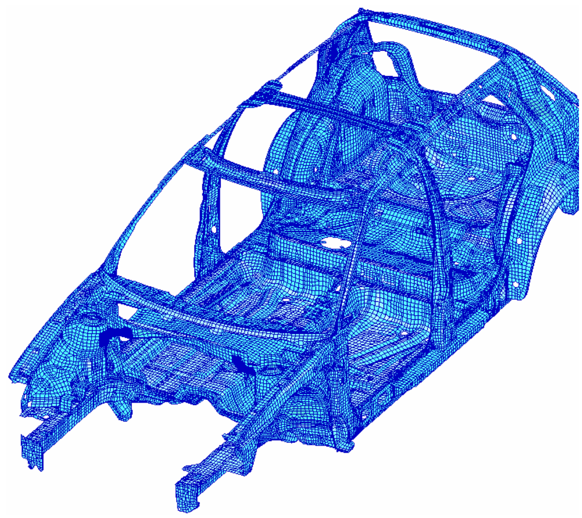

Figure 1: FE model of a car body

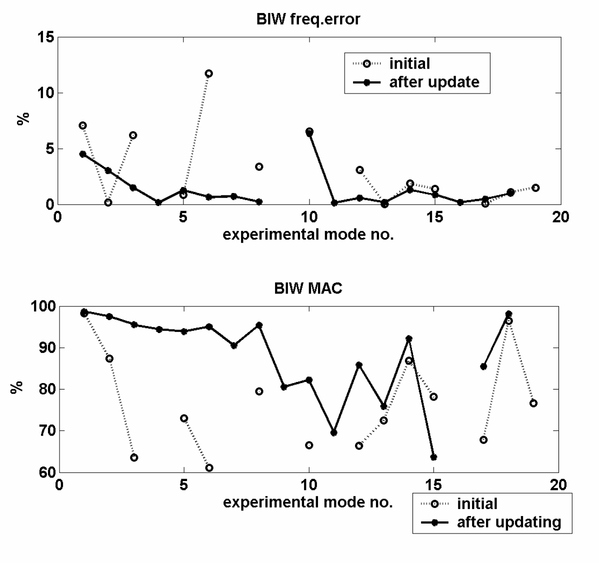

Figure 2: Eigenfrequency deviations and MAC values before and after updating of a model of a car body

3.2 Civil Aircraft

A second large scale application is related to the validation of the FE-model of an aircraft structure as shown in fig.3. Deviation of analytical and experimental results is most often unavoidable due to the complexity of aircraft structures. The need to validate the analytical model is required within the aeroelastic stability certification process of civil aircraft. The analytical models have to satisfy high standards with regard to flutter calculations and windmilling certification calculations. Computational model updating of a large civil aircraft FE-model, ref. [12], are summarized in this lecture. After updating the results correlate well with the data from ground vibration test in the frequency range which was used in the objective function (active frequency range). The model's prediction capability was also checked by comparing the analytical model predictions in the passive frequency range as shown in fig.4 and also to predict the frequency response which was not included in the objective function.

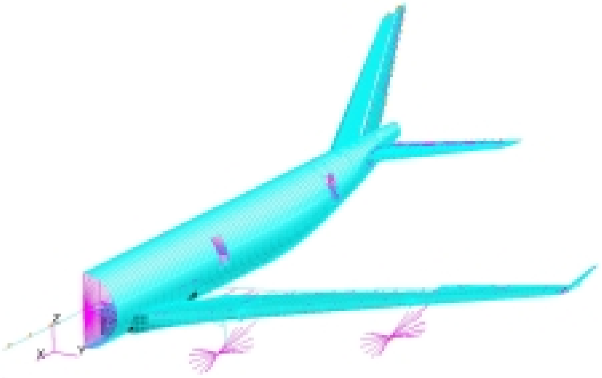

Figure 3: FE- model of 4-engine civil aircraft

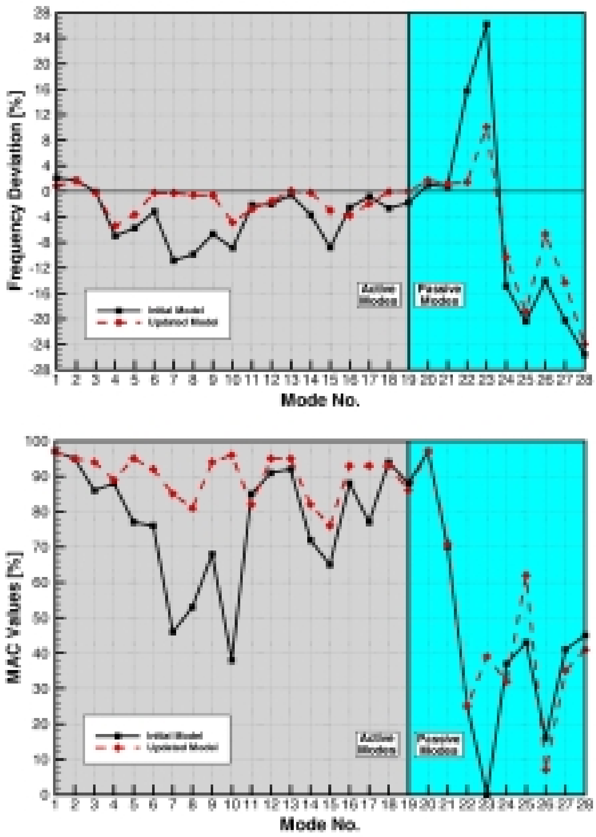

Figure 4: Eigenfrequency deviations and MAC values before and after updating of an aircraft structure

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